

Game Theory Behaves

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Editors' Note: Almost everybody who has taken a basic course in negotiation in the last 20 years has encountered basic game theory, at least to the extent of the widely used "prisoner's dilemma" games. But game theoreticians have been hard at work, and have come up with some disturbing findings that go way beyond the simple strategic calculations in the prisoner's dilemma game and its equivalents. Sally and Jones analyze what has been discovered, and what it means for negotiators who need to think at least one step ahead of their counterpart and one step beyond their own biases. And if you're a real negotiator, you're tough enough not to be scared off by a mere equation or two.

Half a century ago, bargaining was central to the maturation of game theory, a field that uses mathematical theories and laboratory experiments to study strategic interaction. John Nash developed his beautiful bargaining solution by making "certain idealizations" about negotiations, namely, "that the two individuals are highly rational, that each can accurately compare his desires for various things, that they are equal in bargaining skill, and that each has full knowledge of the tastes and preferences of the other."¹ Nash translated these idealizations into simple mathematics. First, accurate comparison of desires allows each bargainer's preferences to be represented by a utility function, u_1 or u_2 , respectively. Second, each bargainer has a threat point, the outcome if no deal occurs—in current negotiation parlance, the best alternative to a negotiated agreement, $BATNA_1$ and $BATNA_2$. The solution to the negotiation is the contract that maximizes the following multiplicative quantity:

$$(u_1 - BATNA_1) \cdot (u_2 - BATNA_2).$$

More important, Nash demonstrated that his idealizations could produce a solution in not just the bargaining game but all other games as well.² The Nash equilibrium is a pair of strategies in a two-player game that are the best possible responses to each other. For example, there is one Nash equilibrium in the matching pennies game. Two players have a penny and must decide which face to show. One player wins if the same face (head or tail) is shown; the other wins if there is a mismatch. The stable pair of strategies consists of each player flipping his or her coin so that the presented face is chosen randomly.

Of course, Nash understood how important his idealized assumptions were to his proof. With respect to whether his model matched the reality of the bargaining

game, he wrote, "The usual haggling process is based on imperfect information, the hagglers trying to propagandize each other into misconceptions of the utilities involved. Our assumption of complete information makes such an attempt meaningless."³ A greater part of the history of game theory over the last half century involves the analysis of whether hagglers, or just perfect players, will arrive at Nash's solution. It happened that many players in laboratories or real markets chose the strategic equivalent of "tails" when Nash's solution predicted "heads." Such mismatching has led to the rise of behavioral game theory, which assumes haggling and other player imperfections are meaningful.

The purpose of this essay is to review several of the new matches between theory and reality that behavioral game theory has been responsible for in the last decade, especially those concurrences that have relevance to those who teach, study and practice the bargaining game. One historical mismatch that we assume most readers are familiar with occurred in the testing of the prisoners' dilemma.⁴ This game has served as an exemplar of the tension between cooperation and competition, between self-interest and joint maximization. Since this game and, to a lesser extent, the ultimatum game have been widely discussed and employed in the negotiation literature, we will focus on games that are not as widely known and on newly identified motives for cooperative or fair behavior.

New Games, New Motives

Strategic Sophistication

One of the primary pieces of advice offered to negotiators is to prepare, prepare, prepare, just like the Boy Scouts, only more. The negotiator is told to consider not only her own interests and issues, but also those of her opponent. Yet, there is a basic question that is almost never addressed: should I prepare for a prepared opponent or an unprepared opponent? This question and its more complicated variants (prepare for a prepared opponent who knows I am preparing?) involve the issue of strategic sophistication. A high degree of strategic sophistication was inherent in Nash's idealizations: his equilibrium arises from two very rational players who choose strategies that are reciprocally best responses to each other. However, if one player is boundedly rational and not really thinking things through, the other's best response to this naïvete might be quite different from the Nash equilibrium strategy.

A clever new guessing game that can diagnose strategic sophistication was introduced in 1995.⁵ The standard numerical guessing game involves a group of players trying to come closest to a target integer between zero and one hundred that someone has picked. This someone might be, for example, a third grade teacher deciding which student gets to take the class bunny home and care for it over vacation week. Here, in Nagel's version, the target number is generated by the players themselves—it is some positive fraction, greater than zero and less than one, of the average of all of their guesses. The person closest to this shared mean wins a prize. Players need to anticipate where the average guess will be and then adjust downward, a cognitive process that depends on making some assumptions about the other players, their understandings of the game and the numbers they are likely to advance.

As a numerical example, suppose there are ten players whose guesses are equally spaced between 0 and 90, i.e., 0, 10, 20, 30, etc. Suppose, also, that the target is $\frac{1}{2}$ of the mean. Then, the average guess is 45, the target is $22\frac{1}{2}$, and the person who guessed 20 would win. Note that the person who guessed 90 is not even close. If you held everybody else's guesses fixed, then she would much prefer

to change her guess to 19.⁶ Now, the person guessing 80 would like to slash her guess, and so on. It turns out that the only Nash equilibria consist of each player guessing 1 or each guessing 0.

Because of the structure of the game, a player's guess reveals how sophisticated she believes the other players are. Assume, for the moment, that all the players except for one are not even bothering to think through the game. We can call them zero-step players because they refuse to enter into the strategic domain, foregoing any consideration of what might be the best move. If our tenth player was strategic, she would choose the best response to her naïve co-participants. This one-step player would select 24,⁷ and would win the game if her prediction about the other players comes true. A two-step player would make a best response to opponents who are all one-step players—a guess of 11.⁸ Three-step players would assume that all others are two-step and would choose the corresponding strategy. The process continues and converges on Nash's idealized players who are infinitely sophisticated and choose 0 or 1. Note that this progression makes the number line diagnostic, as only zero-step players will make guesses significantly greater than 25, one-step players will be around 25, two-step players will cluster around 11, etc.

Experimental tests have revealed how far real players are from Nash's archetypes: the strong majority of participants are either one-step or two-step players.⁹ The remainder are more likely to be zero-step than three-step or more. Other games and experimental technologies have confirmed this finding of modest strategic sophistication.¹⁰ The best place to be in these games, all else being equal, is close to the two-step players, adjusting upward if the game is complicated and more zero-step players are anticipated, or downward if more two-step players are forecasted. In the example game above, that position might be a guess of 13 or 14.

These behavioral game theory results provide a foundation for training in negotiation. Many bargainers are one-step strategists, worried only about their own interests and outcomes and incurious about those of the other side. Much of the knowledge imparted in the classroom is designed to make students be two-step strategists. The prescriptive rule is that you want to be one degree more strategically sophisticated than your counterparts, for it can be just as costly to over-think a negotiation as to under-think it (5 is as bad a guess in the example game as 25 is). "Plan, plan, plan" may be too much; "plan plus one," i.e., go one step further than your opponent, may be just right. In addition, there is a good chance that your counterpart is a one-step player, and therefore, you will need to directly educate him or her about your interests and issues. The counterpart's reticence and lack of inquiry may be due to ineptitude rather than strategy.

Learning

As one might imagine, if these guessing games are repeated with the same players, all the guesses drop pretty quickly to the Nash equilibria.¹¹ Players are effective and quick learners in this type of game, but there are others in which the players never quite figure things out. Learning, both the speed and process of knowledge acquisition and its strategic implementation, has been a very active topic in game theory. Theories and experiments investigate whether players use history to alter their assumption about other players and then optimally respond to these changed expectations, or use it simply to mimic strategies that won in earlier rounds.¹² Note that the first approach, "fictitious play," takes more cognitive effort than the second, "reinforcement." A hybrid model that allowed for both learning approaches depending on the characteristics of the individual and the game was able

to explain the evolution of strategic choices by many players in a wide range of repeated games.¹³

This hybrid model was predictive in part because it allowed sophisticated players both to learn and to anticipate that others were learning as well.¹⁴ Consider the guessing game for a last time, and suppose the target ($\frac{1}{2}$ of the mean) for the first round turned out to be 13. Pure reinforcement learners might choose 13, naïve fictitious play learners would choose 6 or 7, but more sophisticated learners would anticipate the learning of the other players and adjust their strategy accordingly. Camerer writes, “[A]s players gain experience with the game, the degree of sophistication rises—they learn that others (like themselves) are learning.”¹⁵

Negotiation scholars, because of their familiarity with the prisoners’ dilemma, are well versed in the effects of repetition on creating value and encouraging cooperation. They are less attuned to the learning that takes place across repetitions. Simply put, experienced negotiators themselves and the learning processes they employ have been ignored. [Peppet & Moffitt, *How to Learn*] On the experimental side, the reason for this is due to the pools of the usual subjects—students enrolled in a negotiation class, and those from the larger campus. The former are rarely, if ever, confronted with the same negotiation from an earlier week, and the latter are generally quite inexperienced negotiators.

Negotiation should follow the lead of game theory and place learning near the top of its research agenda. The relationship between learning and complexity and the relative importance of reinforcement and fictitious play lead to many fascinating questions that could be studied more rigorously and are quite relevant for the practice of negotiations:

- If a negotiator has success with a particular tactic, how likely is she to use that tactic in the next negotiation? Conversely, if a counterpart has succeeded with a tactic, under what conditions are you likely to trot out the same tactic, or, go one step deeper and introduce a counter-tactic?
- What is the most effective way to teach key principles such as interest-based bargaining? Is there a path or process that is clearly to be preferred?
- Does the Nash bargaining solution appear more frequently when negotiations are recurring? Does this kind of convergence cut the costs of bargaining?
- How much is experience worth? What is the relative cost in the short run and the return on the investment in the long run of sending a neophyte negotiator to the bargaining table?

Social Preferences

A basic principle of negotiation, one that is learned through both reinforcement and the negative effects of its absence, is trust. The trust game was developed in the last ten years and has been employed as another tool to examine the factors of cooperation, reciprocity, fairness, and generosity that the prisoners’ dilemma and ultimatum games have traditionally illuminated.¹⁶ The trust game is a two-person bargaining game that is played as follows: player P is given a certain amount of money, say, \$10. P may give some portion of the endowment to the other player, the receiver, R. Every dollar that P sends to R is doubled or tripled. R, then, makes another allocation decision—how much of the newly augmented pot will be remanded to P. (This basic game is varied by constraining the options for the amounts offered to R and back to P). As is true in its companion games, the trust game rarely results in the uncooperative, untrusting Nash equilibrium of no money being sent in either direction. Rather, positive amounts are usually sent and reciprocated, with the mean and median being around half of the total.¹⁷

The unmistakable implication of these results to behavioral economists has been that individuals are endowed with social preferences, not with the atomistic, self-concerned preferences traditionally assumed in economics.¹⁸ “Full knowledge of the tastes and preferences of the self (and vice versa)” reveals that the other places some weight on the utility of the self (and vice versa). Nash’s bargaining solution is transformed: $(u_1 + \lambda_{12}u_2 - \text{BATNA}_1) \cdot (u_2 + \lambda_{21}u_1 - \text{BATNA}_2)$, with λ_{ij} representing the weight player i places on player j ’s utility. Although such weights are an idea with deep roots in economics going back to Adam Smith and Alfred Marshall, it is only recently that formal other-concern has moved from the margins to a central object of study.¹⁹

Behavioral game theorists have rediscovered the importance and malleability of intentions. A willingness to trust the other party and the evaluation of an offer as fair depend critically on our perception of the other’s intentions.²⁰ In the trust game, for example, if player P publicly forgoes a lucrative option, such as a free agent contract with another employer, receiver R is much more likely to be generous than if P had no choice.²¹ The reason, of course, is that R credits P with good intentions in the first case but not in the second. In general, the utility weight, λ_{ij} , is negative if player j has bad intentions and deserves to be punished, but it is positive if player j is credited with good intentions and deserves to be rewarded.

The perception of intentions, like all perceptions, is ultimately subjective. It is influenced by the personality of the perceiver, the particulars of the social interaction, and the norms and rules of the greater society. One measure of personality is social values orientation, which is disclosed through a series of outcome choices involving various payoffs for self and other.²² Those who are identified as “prosocial” in this test tend to be more cooperative in a variety of games and are more productive in integrative negotiations.²³ More importantly, if the players are physically or psychologically close, prosocial behavior is much more likely.²⁴ The chance to make eye contact, co-presence in a room, shared opinions and attitudes, similarity of appearance and tastes, positive mood and affection all make trust more likely and reliably boost the utility weight of working with the other side (λ_{ij}).²⁵

One study of the trust game allowed participants to meet each other and identify commonalities before choosing, and these participants sent significantly larger offers than did the anonymous, distant subjects of other experiments.²⁶ [Bhappu & Barsness, *E-mail*] Finally, the norms of society serve as a basis for the perception of intentions and the appropriate ways to react to benign or malign intent. For example, a trust game played across segments of Israeli society discovered that all segments were equally unlikely to trust a receiver whose family emigrated from Africa or Asia (Sephardic Jews).²⁷ Despite the fact that these Israelis remitted as large a proportion as did those whose families originated in America or Europe (Ashkenazi Jews), they were entrusted with, on average, only half of the amount that the “Western” Israelis were, and far more “Easterners” than “Westerners” were given nothing at all. The most parsimonious explanation is that the norm or cultural bias in Israel considers “Easterners,” wrongly, to be unreliable and ungenerous.

That economists have finally recognized social preferences may be greeted by negotiation researchers with a chorus of “it’s about time” and “so what?” Nevertheless, this belated “discovery” does present some challenges and opportunities, if only to respond to economists’ innate desire to model and measure as much as they can. Clearly, the process of negotiation may alter social preferences and raise or lower the utility weight (λ_{ij}): the questions are, how much? how often? at

what cost? For example, schmoozing can be thought of as the exchange of trivial personal information with the goal to find salient and public similarities that, in turn, will foster trust, e.g., “you were in Des Moines last week? Oh, my cousin’s best friend’s mother is from there.” How much value is there in schmoozing, physical co-presence, familiarity, and other factors that narrow social distance?

Because we actively, and sometimes unconsciously, participate in the preservation of our perceptions and preferences, situations of great conflict and social distance are especially troublesome. We perceive our enemies to be evil, distant, strange, unapproachable, unfamiliar, distasteful, and unknowable.²⁸ Moreover, we actively resist any evidence to the contrary. Hence, productive negotiations must take a great deal of time, and the process will necessarily have to decrease social distance slowly and imperceptibly.

Finally, there is renewed emphasis on the active management and manipulation of intentions. The following tactics are examples: letting the other side know that a valuable option was foresworn in order to bargain, demonstrating good faith by sharing information early on, apologizing [Brown & Robbennolt, *Apology*] for any wrongs perceived as intentional or excusing them as inadvertent errors, and watching your wallet when someone is overaggressive about narrowing social distance.

The Value of Information

Also inherent in Nash’s idealizations was complete, or perfect, information about the taste and preferences of the bargainer on the other side of the table. As noted earlier, Nash acknowledged that the “usual haggling process”²⁹ was mired in imperfect information, producing an informational asymmetry that has been the focus of a great deal of experimental game theory.³⁰ The mismatch between the perfect predictions of formal models and the imperfectness of actual bargaining interactions has produced a prescription that recommends identification and clarification of relevant party interests (i.e., $u_2 + \lambda_{21}u_1$) and improved estimates of other parties’ alternatives to a negotiated agreement (i.e., $BATNA_2$). To the extent that a party falls short of Nash’s perfect ideal, one would think that they would surely place value on the means to improve their knowledge (even if they decide to apply it in an unsophisticated way as discussed above).

These means are varied and could include additional research effort, external consultants, or “neutral” third parties engaged to guide the negotiation process, offering reality checks where unrealistic party expectations warrant and providing a form of social lubricant to counteract the informational friction that threatens to preserve asymmetry. While there has been significant debate about the extent to which third parties should implicitly or explicitly act to address such imbalances,³¹ it seems clear that if third parties were not able to shift either the utility function or the BATNA of at least one party, then they could have no effect on the outcome of the negotiation.³²

If information is power, if parties inevitably fall short of the Nash idealization of complete information, and if a source exists that can move one closer to the ideal, then how might one put a value on this supplier of advantage? One strategy would be to consider the value of theoretically perfect information—information in fact so probative that the negotiator can proceed with certainty.³³ Knowing the precise interests of the other party, the value of their threat point, or BATNA, and consequently their reservation price, such an omniscient negotiator would be able to maximize value in the negotiation and then claim the maximum share of this value, that share that just meets the other party’s reservation and prevents them from walking away from the table. While such perfect information is rarely avail-

able in practice, it turns out that this theoretical benchmark, known as the expected value of perfect information,³⁴ is fairly straightforward to compute.

Again, however, real parties fall short of even this patched up version of Nash's beautiful bargaining solution. Under experimental conditions, not only do parties suffer the consequences of imperfect information, but they demonstrate a robust tendency to undervalue opportunities to improve (e.g., by participating in a third-party-guided process).³⁵ Indeed, even when uncertainty regarding information sources is incorporated into the model (the expected value of imperfect information), parties still dramatically underestimate the value of these sources.³⁶ And the news gets worse.

Once parties have invested in knowledge that is at least more complete, for example by buying research, hiring consultants, [Wade, *Experts*] or by engaging in a third-party process, the information provided by these sources is poorly utilized. When compared to the rational, Nash-idealization-like, benchmark for information integration, Bayes' theorem,³⁷ parties under experimental conditions are significantly anchored by their initial subjective expectations and are unwilling to modify these expectations, even in the face of reliable information to the contrary.³⁸ Third parties, research and other information sources are both undervalued and underutilized.

Conclusion

So it appears that reality is a bit more complicated than Nash's idealizations might have suggested. Here we have shown the difficulties of accounting for the strategic sophistication in the course of preparation; the challenges inherent in not only learning in repeated play, but in learning that your opponents are learning as well; the complexities of transforming Nash's bargaining solution to include social preferences as well as self-interest; and the tendency to undervalue alternative sources of information once shortfalls from the perfect ideal are recognized. At first this may seem disappointing, and the departures from idealization overwhelming. However, in our view, behavioral game theory offers good news. Many behavioral effects are being replicated in vastly different experimental settings, across gender, culture, and geographical boundaries. Robust effects offer prescriptions, and prescriptions offer tangible improvements for those that teach, study and practice the bargaining game. Game theory works best when game theory behaves.

Endnotes

¹ John Nash, *The Bargaining Problem*, 18 *ECONOMETRICA* 155 (1950).

² See John Nash, *Two-Person Cooperative Games*, 21 *ECONOMETRICA* 128 (1953).

³ *Id.* at 138.

⁴ For a review of experimental results, see David Sally, *Conversation and Cooperation in Social Dilemmas: A Meta-Analysis of Experiments from 1958 to 1992*, 7 *RATIONALITY & SOCIETY* 58 (1995).

⁵ See Rosemarie Nagel, *Unraveling in Guessing Games: an Experimental Study*, 85 *AMERICAN ECONOMIC REVIEW* 313 (1995).

⁶ The solution to the equation that sets one half of the new mean equal to her guess, $x: 1/2 * (360+x)/10=x$.

⁷ Because the nine zero-step players would be choosing randomly, the expected total of their guesses is 450. The one-step player solves the following equation to determine her best guess, $x: 1/2 * (450+x)/10=x$.

⁸ The solution to $1/2 * (216+x)/10=x$.

⁹ See Nagel, *supra*, note 5; Teck-Hua Ho, et al., *Iterated Dominance and Iterated Best Response in Experimental "p-Beauty Contests"*, 88 *AMERICAN ECONOMIC REVIEW* 947 (1998).

¹⁰ See Miguel Costa-Gomes, et al., *Cognition and Behavior in Normal-form Games: An Experimental Study*, 69 *ECONOMETRICA* 1193 (2001); Trey Hedden & Jun Zhang, *What Do You Think I Think You Think? Strategic Reasoning in Matrix Games*, 85 *COGNITION* 1 (2002); Dale O. Stahl &

Paul Wilson, *On Players' Models of Other Players: Theory and Experimental Evidence*, 10 GAMES & ECONOMIC BEHAVIOR 218 (1995).

¹¹ See Ho, et al., *supra* note 9.

¹² See Yin-Wong Cheung & Daniel Friedman, *Individual Learning in Normal Form Games: Some Laboratory Results*, 19 GAMES & ECONOMIC BEHAVIOR 46 (1997); Ido Erev & Alvin Roth, *Predicting How People Play Games: Reinforcement Learning in Experimental Games with Unique, Mixed-Strategy Equilibria*, 88 AMERICAN ECONOMIC REVIEW 848 (1998).

¹³ See Colin Camerer & Teck-Hua Ho, *Experience-Weighted Attraction Learning in Normal Form Games*, 67 ECONOMETRICA 827 (1999).

¹⁴ See Colin Camerer, et al., *Sophisticated Experience-Weighted Attraction Learning and Strategic Teaching in Repeated Games*, 104 JOURNAL OF ECONOMIC THEORY 137 (2002).

¹⁵ Colin Camerer, *Behavioral Studies of Strategic Thinking in Games*, 7 TRENDS IN COGNITIVE SCIENCE 225, 227 (2003).

¹⁶ See Joyce Berg, et al., *Trust, Reciprocity and Social History*, 10 GAMES & ECONOMIC BEHAVIOR 122 (1995).

¹⁷ *Id.*; See Kevin McCabe, et al., *Reciprocity, Trust and Payoff Privacy in Extensive Form Bargaining*, 24 GAMES & ECONOMIC BEHAVIOR 10 (1998); Andreas Ortmann, et al., *Trust, Reciprocity and Social History: a Re-examination*, 3 EXPERIMENTAL ECONOMICS 81 (2000).

¹⁸ See David Sally, *A General Theory of Sympathy, Mind-Reading, and Social Interaction with an Application to the Prisoners' Dilemma*, 39 SOCIAL SCIENCE INFORMATION 567 (2000); Gary Charness & Matthew Rabin, *Understanding Social Preferences with Simple Tests*, 117 QUARTERLY JOURNAL OF ECONOMICS 817 (2002).

¹⁹ See ADAM SMITH, *THEORY OF MORAL SENTIMENTS* (1790); ALFRED MARSHALL, *THE EARLY ECONOMIC WRITINGS OF ALFRED MARSHALL, 1867-1890*, VOL. 2 (John K. Whitaker ed., 1975). See also, Sally, *supra* note 18, for a review of the history.

²⁰ See Sally, *supra* note 18; Matthew Rabin, *Incorporating Fairness into Game Theory and Economics*, 83 AMERICAN ECONOMIC REVIEW 1281 (1993).

²¹ See Kevin McCabe, et al., *Positive Reciprocity and Intentions in Trust Games*, 52 JOURNAL OF ECONOMIC BEHAVIOR & ORGANIZATION 267 (2003).

²² See David Messick & Charles McClintock, *Motivational Basis of Choice in Experimental Games*, 4 JOURNAL OF EXPERIMENTAL SOCIAL PSYCHOLOGY 1 (1968).

²³ See Carsten De Dreu, et al., *Influence of Social Motives on Integrative Negotiations: a Meta-Analytic Review and Test of Two Theories*, 78 JOURNAL OF PERSONALITY & SOCIAL PSYCHOLOGY 889 (2000).

²⁴ See Sally, *supra* note 18; Elizabeth Hoffman, et al., *Social Distance and Other-Regarding Behavior in Dictator Games*, 86 AMERICAN ECONOMIC REVIEW 653 (1996).

²⁵ The complete argument including connections to the literature in social and cognitive psychology is contained in Sally, *supra* note 18.

²⁶ See Edward Glaeser, et al., *Measuring Trust*, 115 Q. J. ECON. 811 (2000).

²⁷ See Chaim Fershtman & Uri Gneezy, *Discrimination in a Segmented Society: An Experimental Approach*, 116 QUARTERLY JOURNAL OF ECONOMICS 351 (2001).

²⁸ See David Sally, *Into the Looking Glass: Discerning the Social Mind Through the Mindblind*, 18 ADVANCES IN GROUP PROCESSES 99 (2001).

²⁹ See Nash, *supra* note 1.

³⁰ See Camerer, *supra* note 15.

³¹ A debate that will not be revisited here. For a complete coverage of concerns on both sides of the issue, see Gregory Jones & Douglas Yarn, *Evaluative Dispute Resolution Under Uncertainty: an Empirical Look at Bayes' Theorem and the Expected Value of Perfect Information*, 2003 JOURNAL ON DISPUTE RESOLUTION 427 (2003).

³² Many forms of alternative dispute resolution "share the feature that a third party is involved who offers an opinion or communicates information about the dispute to the disputants"—information that should cause the parties to modify their subjective expectations regarding possible outcomes. See Steven Shavell, *Alternative Dispute Resolution: an Economic Analysis*, 24 JOURNAL OF LEGAL STUDIES 1 (1995); see also, Gregory Jones, et al., *Evaluative Dispute Resolution Under Uncertainty: Framing, Confirmatory Evidence Bias, and the Expected Value of Imperfect Information* (unpublished working paper) (on file with author).

³³ See Jones, *supra* note 32.

³⁴ For a thorough, but accessible, treatment of the mathematics, see ROBERT CLEMEN, *MAKING HARD DECISIONS: AN INTRODUCTION TO DECISION ANALYSIS* (1991).

³⁵ See Jones, *supra* note 32.

³⁶ *Id.*

³⁷ *Id.*

³⁸ *Id.*